# WIENER RECONSTRUCTION OF THE IRAS 1.2Jy GALAXY REDSHIFT SURVEY: COSMOGRAPHICAL IMPLICATIONS

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#### Abstract

We utilise the method of Wiener reconstruction with spherical harmonics and Bessel functions to recover the density and velocity fields from the IRAS 1.2Jy redshift survey. The reconstruction relies on prior knowledge of the IRAS power-spectrum and the combination of density and bias parameters  $\beta \equiv \Omega^{0.6}/b$ . The results are robust to changes in these prior parameters and the number of expansion coefficients. Maps of these fields are presented in a variety of projections. Many known structures are observed, including clear confirmations of the clusters N1600 and A3627. The Perseus-Pisces supercluster appears to extend out to  $\sim 9000 \; \mathrm{km \; s^{-1}}$ , and the reconstruction shows 'backside infall' to the Centaurus/Great Attractor region. A qualitative comparison of the reconstructed IRAS gravity field with that from Tully-Fisher peculiar velocity measurements (Mark III) shows reasonable agreement. The Wiener reconstruction of the density field is also the optimal reconstruction (in the minimum variance sense) of any quantity which is linear in the density contrast. We show reconstructions of three such quantities. The misalignment angle between the IRAS and CMB Local Group dipoles is only  $13^{\circ}$  out to  $5000 \text{ km s}^{-1}$ , but increases to  $25^{\circ}$  out to  $20,000 \text{ km s}^{-1}$ . The reconstructed IRAS bulk flow out to 5000 km s<sup>-1</sup> is  $\sim 300$  km s<sup>-1</sup>, which agrees in amplitude with that derived from the Mark III peculiar velocities ( $\sim 370 \text{ km s}^{-1}$ ). However, the two bulk flow vectors deviate by  $\sim 70^{\circ}$ . Finally, moment of inertia analysis shows that the Wiener reconstructed Supergalactic Plane is aligned within  $\sim 30^{\circ}$  of that defined by de Vaucouleurs.

# 1. Introduction

Observations of large scale structure have been advancing spectacularly. Both in terms of redshifts and galaxy peculiar velocities, the number of measurements has grown by more than a factor of ten over the past decade. Hand in hand with observational progress, a plethora of statistical methods has been developed to analyse the data, each attempting to extract as much astrophysical information as possible. Accurate determination of the galaxy distribution and velocity field would not only place constraints on cosmological parameters, but would also provide an insight into the mechanisms of structure formation which generated the complex pattern of sheets and filaments we observe today.

Redshifts have been determined for many more galaxies than those for which we have direct distance measurements. Over the next few years, automated surveys such as the 2-degree-Field (2dF) and Sloan Digital Sky Survey (SDSS) will increase this lead even further, as over a million new galaxy redshifts become available. This wealth of data has inspired a great deal of work on techniques for reconstructing the velocity field on the basis of redshifts alone. Given the assumptions of linear mass-to-light biasing, and purely linear

structure evolution, the mapping between real and redshift-space is well defined (Kaiser 1987). However, in applying reconstruction methods there is freedom of choice regarding (i) the functional representation (e.g. Cartesian, Fourier, Spherical Harmonics, or Wavelets) and (ii) the filtering, or smoothing scheme (e.g. a Gaussian sphere, a sharp cut-off in Fourier space, or a Wiener filter). Smoothing is necessary to reduce sampling noise before the real-space density field can be calculated from that in redshift-space.

The first reconstruction technique to be applied to redshift surveys was based on iterative solution of the equations of linear theory, pioneered by Yahil et al. (1991), hereafter YSDH (see also Yahil 1988; Strauss & Davis 1988). This involves solving for the gravity field in redshift space, which can then be used to derive an estimate of the peculiar velocities for a given value of  $\beta \equiv \Omega^{0.6}/b$ . These velocities allow the redshifts to be corrected, providing an updated set of distance estimates. This is repeated until the distance estimates converge. YSDH use variable smoothing, but their smoothing regime is not rigorously formulated. Variants of this technique have been successfully applied to IRAS selected galaxy catalogues (YSDH; Kaiser et al. 1991).

The reconstruction procedure used in this paper follows Fisher *et al.* (1995a; hereafter FLHLZ) and is *non*-iterative. The density field in redshift-space is expanded in terms of spherical harmonics and Bessel functions. The real-space density, velocity and potential fields are reconstructed using linear theory and a Wiener filter which assumes a given power spectrum and noise level. It is important to note that, as opposed to ad-hoc smoothing schemes, the smoothing due to a Wiener filter is determined by the sparseness of data relative to the expected signal.

Cosmography using spherical harmonics is not a new technique (Peebles 1973), but it has enjoyed a renaissance with the advent of near whole-sky galaxy surveys (e.g. Fabbri & Natale 1989; Regos & Szalay 1989; Lynden-Bell 1991; Scharf et al. 1992; Scharf & Lahav 1993; Fisher, Scharf & Lahav 1994, hereafter FSL; FLHLZ; Lahav 1994; Nusser & Davis 1994; Heavens & Taylor 1995). Similarly, the Wiener filter (Wiener 1949; hereafter WF) has long been used in engineering to recover the best estimate of the true signal from one corrupted by imperfect measurement. Recently, Wiener filtering has been applied to a number of cosmological reconstruction problems. Lahav et al. (1994; hereafter LFHSZ) reconstructed the angular distribution of IRAS galaxies, while Bunn et al. (1994) applied the method to the temperature fluctuations in the COBE DMR maps. Kaiser & Stebbins (1991) used a similar method to reconstruct velocity fields. Reviews of the WF and linear estimation methods can be found in Press et al. (1992), Rybicki & Press (1992) and Zaroubi et al. (1995; hereafter ZHFL).

FLHLZ applied the 3-dimensional Wiener reconstruction to the IRAS 1.2Jy survey, showing preliminary predictions for the density, velocity and potential fields. In this paper, we expand on the initial results in FLHLZ, presenting detailed maps of the reconstructed fields, as well as optimal determinations of the Local Group dipole, bulk flows and the extent of the Supergalactic Plane. We compare the IRAS reconstructed peculiar velocity field to that derived from Tully-Fisher distances (Mark III) and examine the sensitivity of the WF method to choice of parameters.

In §2, we give a brief summary of the decomposition and filtering techniques used in this reconstruction and the data set to which they have been applied. §3 discusses the role of the prior model in the WF and the methodology behind selection of parameters. Results of the reconstruction are shown in §4, and these are compared to Mark III data in §5. As detailed below, the choice of prior model is central to the operation of the WF and §6 discusses the qualitative influence of these parameters on the reconstructed density and velocity fields. In §7 and §8, we show reconstructions of the Local Group dipole and bulk flow respectively; both determined directly from harmonic coefficients. Similarly, a reconstruction of the Supergalactic Plane is shown in §9, and its alignment compared to previous measurements. Finally §10 contains a discussion of the work in this paper.

## 2. Wiener reconstruction of IRAS data

#### 2.1 Reconstruction Method

This paper concentrates on cosmography, presenting results from the application of the WF method to IRAS 1.2Jy survey data. Hence, we include here only a brief introduction to our reconstruction technique. The approach we use is discussed in detail by FLHLZ, while the formalism of Wiener filtering as it pertains to LSS reconstruction can be found in ZHFL.

A density field,  $\rho$ , can be expanded in terms of spherical harmonics,  $Y_{lm}$ , and Bessel functions,  $j_l$ :

$$\rho(\mathbf{s}) = \sum_{l=0}^{l_{\text{max}}} \sum_{m=-l}^{+l} \sum_{n=1}^{n_{\text{max}}(l)} C_{ln} \, \rho_{lmn} \, j_l(k_n s) \, Y_{lm}(\hat{\mathbf{s}}) \qquad . \tag{1}$$

The discrete  $k_n$ 's are chosen according to the boundary conditions, so as to make the set orthogonal (see Appendix A in FLHLZ). This process is analogous to Fourier decomposition, but instead using a set of spherical basis functions. The data from a redshift catalogue can be seen as a set of N discrete points,  $\mathbf{s}_i$ , each giving the direction and redshift of a galaxy. These are used to estimate the underlying density field in redshift space,  $\hat{\rho}^{S}(\mathbf{s})$ , expanded as in equation 1. Here,  $C_{ln}$  are normalisation constants, while the harmonic coefficients are given by

$$\hat{\rho}_{lmn}^{S} = \sum_{i=1}^{N} \frac{1}{\phi(s_i)} j_l(k_n s_i) Y_{lm}^*(\hat{\mathbf{s}}_i) \qquad , \tag{2}$$

where  $\phi(s_i)$  is the spherical selection function of the survey, evaluated at the radius of the  $i^{th}$  galaxy. From  $\hat{\rho}(\mathbf{s})$ , we can derive an estimate of the overdensity field,  $\delta(\mathbf{s}) = (\rho(\mathbf{s})/\bar{\rho}) - 1$ , in terms of the same basis functions, with coefficients  $\hat{\delta}_{lmn}^{S}$ . This is not only more useful for dynamics, but also has, by construction, a mean of zero, which is useful later on in the reconstruction method.

As redshifts are line-of-sight measurements, peculiar velocities will introduce only radial distortion in redshift space. Given that  $\delta^{S}$  is expressed in terms of orthogonal radial and angular components, redshift distortion can be seen as coupling between the radial modes in real and redshift space, leaving the angular modes unaffected. This can be expressed by introducing a matrix which links radial modes in the density fluctuation fields,

$$\hat{\delta}_{lmn}^{S} = \sum_{n'} (\mathbf{Z}_l)_{nn'} \,\hat{\delta}_{lmn'}^{R} \qquad , \tag{3}$$

where  $\mathbf{Z}_l$  depends on the chosen value of  $\beta$  and the selection function for the survey (see Appendix D in FLHLZ). Here we use superscripts R and S to denote real and redshift space respectively. Note that this is only applicable for all-sky surveys; in samples with incomplete sky coverage, there will also be more complicated coupling of angular modes.

In a perfect galaxy catalogue, with arbitrarily high sampling, the real-space harmonics could be estimated by simple inversion of the coupling matrix in equation 3. However, with real data, straightforward inversion is often unstable, and a regularisation scheme is required. Without such a scheme, shot noise from the discreteness of the galaxy distribution might be amplified by inversion, leading to an estimate that was far from optimal. Application of a Wiener filter to the inversion process allows one to reconstruct the minimum variance solution, assuming a prior power spectrum and a shot noise level. Appendix A provides a summary of the WF method. For further details, please refer to Rybicki & Press (1992), LFHSZ and ZHFL.

In our problem, the redshift coupling matrix represents the response function of the system, acting on the real-space distribution plus shot noise to give the observed,  $\delta^{S}$ . Hence, the WF estimation of the real space overdensity field is

$$\left(\hat{\delta}_{lmn}^{R}\right)_{\text{wf}} = \sum_{n'n''} \left( S_l \left[ S_l + \mathbf{N}_l \right]^{-1} \right)_{nn'} \left( \mathbf{Z}_l^{-1} \right)_{n'n''} \hat{\delta}_{lmn''}^{S} \qquad . \tag{4}$$

The assumed true signal matrix, S, depends on the power spectrum, while the noise matrix,  $\mathbf{N}$ , is a function of the mean density and selection function (see LFHSZ). Incidentally, as Rybicki & Press (1992) point out,

the WF is in general a biased estimator of the mean field unless the field has zero mean. As mentioned above, the density fluctuation field has zero mean.

Just as the density field can be decomposed into harmonics, so too can the radial and transverse components of the velocity field, and the potential field. In the case of radial velocities, linear theory allows the velocity due to inhomogeneities within r < R to be expressed directly in terms of the harmonics of the overdensity field. Again, this can be expressed by introducing a coupling matrix, in a similar manner to that in equation 3, such that

$$(v_{lmn})_{\text{wf}} = \beta \sum_{n'} (\Xi_l)_{nn'} (\delta_{lmn'}^{\text{R}})_{\text{wf}} \qquad . \tag{5}$$

Transverse velocity and potential harmonics are also related to those of the density field (Appendix C in FLHLZ).

To summarise, the redshift-space density field is estimated in terms of orthogonal functions which are eigenfunctions of the Laplacian operator. For a given choice of  $\beta$ , the radial coupling matrix linking real- and redshift-space harmonics can be determined. Regularisation by a Wiener filter, with a given power spectrum as its prior, allows inversion of the coupling matrix and determination of the real-space density field. Linear theory then allows the velocity and potential fields to be determined from the estimated density field. We emphasise that the method is *non*-iterative, and uses a filter which is optimal in terms of assumed signal and noise characteristics. It provides a non-parametric and minimum variance estimates of the density, velocity, and potential fields which are related by simple linear transformations.

## 2.2 The IRAS 1.2 Jy Survey

The IRAS surveys are uniform and complete down to Galactic latitudes as low as  $\pm 5^{\circ}$  from the Galactic plane. This makes them ideal for estimating whole-sky density and velocity fields. Here, we use the 1.2 Jy IRAS survey (Fisher *et al.* 1995b), consisting of 5313 galaxies, covering 87.6% of sky with the incomplete regions being dominated by the 8.7% of the sky with  $|b| < 5^{\circ}$ .

In principle, the method can be extended to explicitly account for the incomplete sky coverage. We adopt the simpler approach of smoothly interpolating the redshift distribution over the missing areas using the method described in YSDH. Given this, we can assume a purely radial selection function. In LFHSZ, we examined the validity of this interpolation and found its effect on the computed harmonics to be negligible for  $l \lesssim 15$  for the geometry of the 1.2 Jy survey. We therefore use the interpolated catalogue (and a simplified formalism) for our reconstructions.

## 3. Selection of a prior model

Throughout this paper, the density field is expanded within a spherical volume of radius  $R = 20,000 \text{ km s}^{-1}$ . Boundary conditions are imposed at the edge of this volume, forcing the radial modes to be discrete rather than continuous. We have demanded that the logarithmic derivative of the gravitational potential be continuous at the boundary; a discussion of various boundary conditions is contained in Appendix A of FLHLZ. We then truncate the summation over angular and radial modes at some  $l_{\text{max}} = 15$  and  $k_n R < 100$  respectively, effectively determining the resolution of the density field (please see Appendix B of FLHLZ for further details). The qualitative effects of  $l_{\text{max}}$  on resolution are discussed in §6.

Beyond the definition of the expansion, the reconstruction method itself has a number of free parameters. The WF assumes a prior power spectrum, while the redshift coupling matrix depends on choice of  $\beta$ . Fortunately, the shape and amplitude of the power spectrum has been relatively well determined for IRAS galaxies. The power spectrum is described on scales  $\lesssim 200~h^{-1}{\rm Mpcby}$  a CDM power spectrum with shape parameter  $\Gamma$  (see Efstathiou, Bond, & White 1992) in the range  $\Gamma \simeq 0.2-0.3$  (Fisher *et al.* 1993; Feldman, Kaiser, & Peacock 1994). The normalisation of the power spectrum is conventionally specified by the variance

of the galaxy counts in spheres of 8  $h^{-1}{\rm Mpc}$ ,  $\sigma_8$ . Fisher *et al.* (1994*a*) used the projection of the redshift space correlation function to deduce the value,  $\sigma_8 = 0.69 \pm 0.04$  in *real* space. We have adopted the WF prior given by  $\Gamma = 0.2$  and  $\sigma_8 = 0.7$ .

For simplicity, we assume linear, scale-independent biasing, where b measures the ratio between fluctuations in the galaxy and mass distribution,

$$(\delta \rho/\rho)_{\rm gal} = b(\delta \rho/\rho)_{\rm mass} \qquad . \tag{6}$$

In linear theory, the density and velocity fields are related by  $\beta \equiv \Omega^{0.6}/b$ , where  $\Omega$  is the density parameter. The correct value of  $\beta$  for IRAS is uncertain. Strauss & Willick (1995) review work on determining  $\beta_{\rm IRAS}$  by a variety of methods, giving values in the range  $0.35 \leq \beta_{\rm IRAS} \leq 1.28$ .

As discussed in FLHLZ, our reconstruction technique is essentially perturbative, involving a Taylor expansion to first order in  $\Delta \mathbf{v} = (\mathbf{v}(\mathbf{r}) - \mathbf{v}(\mathbf{0}))$ , where  $\mathbf{v}(\mathbf{0})$  is the motion of the observer at the same frame as  $\mathbf{v}(\mathbf{r})$ . Given that the choice of calculation frame is arbitrary, it seems sensible to select a frame in which  $\Delta \mathbf{v}$  is minimal. It is known that nearby galaxies (nearer than  $\sim 2000 \text{ km s}^{-1}$ ) have small peculiar velocities relative to the Local Group (LG), while very distant galaxies are, on average, most likely to be at rest with respect to the Cosmic Microwave Background (CMB). Rather than taking an ad-hoc value for  $\beta$ , it makes sense to try the reconstruction in both frames, and take the value of  $\beta$  for which the two calculations are in best agreement.

We compare the two reconstructed line-of-sight peculiar velocities at the position of all catalogued IRAS galaxies by calculating the RMS scatter as function of  $\beta$ . Figure 1 shows that the minimum scatter occurs at  $\beta \approx 0.7$ ; this result is robust to changes in the limiting radius of the reconstruction. There are many other possible methods for selecting the prior value of  $\beta$ . For example, we can choose the value which best reconstructs the motion of the LG with respect to the CMB; this gives  $\beta \approx 0.6$  to recover the 600 km s<sup>-1</sup>. Another alternative is to find the  $\beta$  for which the Local Group motion is identical in both calculation frame, yielding  $\beta \approx 0.8$ . FLHLZ used  $\beta = 1.0$ , based on maximum likelihood analysis in FSL. Hereafter we adopt  $\beta = 0.7$  based on figure 1, which is well within the range quoted by various studies (Strauss & Willick 1995). In §6 we investigate the effects on the reconstruction of changing  $\beta$ ,  $\Gamma$  and  $\sigma_8$ .

## 4. Density and Velocity Maps

The maps shown in this section were derived by the technique detailed above. In summary, the reconstruction was carried out within a sphere of radius  $R=20,000~\rm km~s^{-1}$ , with angular modes limited to  $l_{\rm max}=15$  and radial modes to  $k_nR<100$ , with a total of 6905 coefficients. A canonical set of values were adopted for cosmological parameters, such that  $\beta=0.7$ ,  $\Gamma=0.2$  and  $\sigma_8=0.7$ . The reconstruction has a variable smoothing scale similar to a Gaussian window of width,  $\sigma_s$ , proportional to the mean inter-particle separation (e.g.  $\sigma_s=436~\rm km~s^{-1}$  at  $r=4000~\rm km~s^{-1}$ ,  $\sigma_s=626~\rm km~s^{-1}$  at 6000 km s<sup>-1</sup>, and  $\sigma_s=1130~\rm km~s^{-1}$  at 10,000 km s<sup>-1</sup>). The density contrast values shown in the diagrams and quoted below have been obtained under this smoothing regime. Figures 2 and 3 show Aitoff projections of the real-space density and radial velocity fields respectively, plotted in Galactic coordinates, evaluated across shells at various distances. Many previous papers have examined parts of the volume shown in these maps, identifying clusters and voids (e.g. Tully 1987, Pellegrini *et al.* 1990, Saunders *et al.* 1991). Where obvious analogues exist, structures are labelled in accordance with previous papers.

Figure 2a shows a density shell at  $r=2000~\rm km~s^{-1}$ . All major features have already been identified, and there is the expected concentration of clusters in the northern Galactic hemisphere. In particular, note Fornax-Doradus-Eridanus ( $l\simeq180^\circ$ ,  $b\simeq-60^\circ$ ), N5846 ( $l\simeq350^\circ$ ,  $b\simeq45^\circ$ ), Virgo ( $l\simeq290^\circ$ ,  $b\simeq70^\circ$ ) and Ursa Major ( $l\simeq135^\circ$ ,  $b\simeq30^\circ$ ). The foregrounds of Hydra ( $l\simeq270^\circ$ ,  $b\simeq10^\circ$ ), and Centaurus ( $l\simeq310^\circ$ ,  $b\simeq20^\circ$ ) are visible, and these structures can be seen to extend onto the next shell. The observed shoulder on Hydra at  $l\simeq240^\circ$ ,  $b\simeq0^\circ$  is due to Puppis (e.g. Lahav *et al.* 1993), which appears as a distinct cluster at

 $r=2500~{\rm km~s^{-1}}$ . The overdensity marked C $\alpha$  ( $l\simeq 10^{\circ}, b\simeq -60^{\circ}$ ) has previously been identified as part of the Pavonis-Indus-Telescopium (P-I-T) supercluster (Santiago *et al.* 1995). However, in this reconstruction, C $\alpha$  appears to be quite distinct, peaking at  $r\simeq 2500~{\rm km~s^{-1}}$  with  $\delta_{\rm max}=2.8$  on the smoothing scale described above. The Local Void (LV), as discussed by Tully (1987) is clearly seen in the generally underdense region roughly spanning 330° <  $l<120^{\circ}, -50^{\circ} < b < 30^{\circ}$ . Two additional voids are also marked, V $\alpha$  (150° <  $l<220^{\circ}, -30^{\circ} < b < 30^{\circ}$ ) and V $\beta$  (240° <  $l<300^{\circ}, -90^{\circ} < b < -20^{\circ}$ ). The LV and V $\beta$  regions extend out to  $r\simeq 4500~{\rm km~s^{-1}}$ , while V $\alpha$  ends at  $r\simeq 3500~{\rm km~s^{-1}}$ .

Figure 2b is density at  $r = 4000 \text{ km s}^{-1}$ . Continuing from the previous shell, the Hydra ( $l \simeq 270^{\circ}$ ,  $b \simeq 25^{\circ}$ ) and Centaurus/Great Attractor (GA;  $b \simeq 315^{\circ}$ ,  $b \simeq 15^{\circ}$ ) superclusters are shown clearly, both extending across  $-10^{\circ}$  < b < 45°. The foregrounds of P-I-T ( $l \simeq 340^{\circ}$ ,  $b \simeq -25^{\circ}$ ) and Perseus-Pisces (P-P;  $l \simeq 150^{\circ}$ ,  $b \simeq -15^{\circ}$ ) can be seen, as can Leo ( $l \simeq 240^{\circ}$ ,  $b \simeq 75^{\circ}$ ), Cancer ( $l \simeq 190^{\circ}$ ,  $b \simeq 25^{\circ}$ ) and Cetus ( $l \simeq 180^{\circ}$ ,  $b \simeq -60^{\circ}$ ). There is also confirmation of N1600 ( $l \simeq 200$ ,  $b \simeq -28$ ), which is the strongest overdensity on this shell ( $\delta_{\text{max}} = 4.2$ ). The position of N1600 closely matches that previously determined in both the IRAS (Saunders et al. 1991) and ORS (Santiago et al. 1995) surveys. Camelopardalis  $(l \simeq 145^{\circ}, b \simeq 30^{\circ})$  is visible in the northern foreground of the Perseus-Pisces supercluster (centre at  $l \simeq 150^{\circ}$ ,  $b \simeq -15^{\circ}$ ). The two are linked by a cluster marked C $\beta$  ( $l \simeq 135^{\circ}$ ,  $b \simeq 10^{\circ}$ ), making a wall of continuous overdensity extending over  $-85^{\circ} < b < 45^{\circ}$ . Extensions of LV and V $\alpha$  are also seen, although the underdensity is considerably less smooth than at  $r = 2000 \text{ km s}^{-1}$ . Three dense regions on the shell do not correspond well to any previous labelled clusters. Cy  $(l \simeq 110, b \simeq -70)$  is the foreground of a structure which extends out to  $r \simeq 8000 \text{ km s}^{-1}$ , including the previously-identified A194 galaxy cluster at  $\sim 6000 \text{ km s}^{-1}$ . C $\delta$  ( $l \simeq 105, b \simeq -40$ ) is the foreground of a cluster which extends to  $r \simeq 6500 \text{ km s}^{-1}$ . where it merges into Pegasus. Finally,  $C\epsilon$  ( $l \simeq 65$ ,  $b \simeq 70$ ) is the foreground of a weakly overdense wall that merges into the Great Wall structure at  $r \simeq 8000 \text{ km s}^{-1}$ .

Figure 2c is density at  $r=6000~\rm km~s^{-1}$ . Continuing from the previous shell, note clear extensions of P-P ( $l\simeq 150^\circ$ ,  $b\simeq -15^\circ$ ), P-I-T ( $l\simeq 0^\circ$ ,  $b\simeq -30^\circ$ ), Hydra ( $l\simeq 280^\circ$ ,  $b\simeq 10^\circ$ ), Leo ( $l\simeq 270^\circ$ ,  $b\simeq 60^\circ$ ), Cy ( $l\simeq 120^\circ$ ,  $b\simeq -70^\circ$ ) and Ce ( $l\simeq 0^\circ$ ,  $b\simeq 70^\circ$ ). Faint backgrounds can also be seen for Cancer ( $l\simeq 190^\circ$ ,  $b\simeq 30^\circ$ ) and Cb ( $l\simeq 110^\circ$ ,  $b\simeq -45^\circ$ ). Orion ( $l\simeq 190^\circ$ ,  $b\simeq 0^\circ$ ), as shown on this shell, is the background of a strong cluster centred at  $r\simeq 5000~\rm km~s^{-1}$ , where it links Cancer and N1600 to form a structure extending over  $-45^\circ$  <  $b<45^\circ$ . A569 ( $l\simeq 165^\circ$ ,  $b\simeq 20^\circ$ ) is centred on this shell, and corresponds very well to the position, extent and redshift (Abell et al. 1989). The foreground of the Pegasus ( $l\simeq 90^\circ$ ,  $b\simeq -20^\circ$ ) supercluster is seen weakly; in this reconstruction, Pegasus extends to  $r>12,000~\rm km~s^{-1}$ . There is also confirmation of the cluster reported by Kraan-Korteweg et al. (1996), marked A3627 ( $l\simeq 120^\circ$ ,  $b\simeq -8^\circ$ ), which can be seen distinctly now that Centaurus no longer swamps the region. The remaining cluster, labelled C $\zeta$  ( $l\simeq 40^\circ$ ,  $b\simeq -20^\circ$ ) does not correspond well to any known labels; it is the foreground of a large structure extending to  $r>10,000~\rm km~s^{-1}$ , and appears to be quite separate from the nearby P-I-T supercluster.

Figure 2d is density at  $r=8000~\rm km~s^{-1}$ . Continuing from the previous shell, P-P ( $l\simeq 150^\circ$ ,  $b\simeq 0^\circ$ ), Leo ( $l\simeq 260^\circ$ ,  $b\simeq 45^\circ$ ), A3627 ( $l\simeq 320^\circ$ ,  $b\simeq -8^\circ$ ) and C $\zeta$  ( $l\simeq 30^\circ$ ,  $b\simeq -35^\circ$ ) extend out to  $r\simeq 8000~\rm km~s^{-1}$ . The cluster marked C $\epsilon$  has merged into the body of the Great Wall (GW; ( $l\simeq 0^\circ$ ,  $0^\circ < b < 90^\circ$ ), which itself merges with C $\zeta$  at  $r\simeq 10,000~\rm km~s^{-1}$  to form a wall extending over  $-60^\circ < b < 90^\circ$ . Another wall is formed by Leo and Coma ( $60^\circ < l < 240^\circ$ ,  $b\simeq 75^\circ$ ); this also seems to extend down beyond the Galactic equator at greater distances. A779 ( $l\simeq 170^\circ$ ,  $b\simeq 45^\circ$ ), A539 ( $l\simeq 190^\circ$ ,  $b\simeq -20^\circ$ ) and A400 ( $l\simeq 170^\circ$ ,  $b\simeq -40^\circ$ ) all correspond to clusters in the Abell catalogue. Cygnus ( $l\simeq 80^\circ$ ,  $b\simeq 15^\circ$ ) and C $\eta$  ( $l\simeq 250^\circ$ ,  $b\simeq -5^\circ$ ) are both foregrounds of structures extending to  $r\simeq 10,000~\rm km~s^{-1}$ . Notably, in this reconstruction, P-P extends out to roughly 9000 km s<sup>-1</sup>; far further than usually thought. Out at  $r>6000~\rm km~s^{-1}$ , the peak overdensity for P-P lies very close to the Galactic equator, and well within the zone of avoidance (ZOA). It is important to remember that C $\eta$  and P-P both lie largely within the  $|b| < 5^\circ$  region in which data has been interpolated to fill the zone of avoidance. However, both are strong overdensities on this shell ( $\delta_{\rm P-P}\simeq 4$ ), and the shoulders are clearly visible in the IRAS data.

Figure 3a shows line-of-sight velocity at  $r = 2000 \text{ km s}^{-1}$ . The major feature in this plot is the velocity

dipole due to the Centaurus/GA region (towards  $l \simeq 300^\circ$ ,  $b \simeq 15^\circ$ ). Also visible are the effects of the P-P/Cancer/N1600 mass concentration, causing outflow towards  $l \simeq 165^\circ$ ,  $b \simeq -10^\circ$ . In figure 3b, at  $r = 4000 \; \mathrm{km \; s^{-1}}$ , some backside infall is visible in the Centaurus/GA region. The strongest features are outflow towards P-P ( $l \simeq 145^\circ$ ,  $b \simeq -20^\circ$ ), N1600 ( $l \simeq 195^\circ$ ,  $b \simeq -20^\circ$ ) and Cancer ( $l \simeq 190^\circ$ ,  $b \simeq 20^\circ$ ), while weaker outflow can also be seem towards the centre of P-I-T ( $l \simeq 340^\circ$ ,  $b \simeq -20^\circ$ ). Figure 3c, at  $r = 6000 \; \mathrm{km \; s^{-1}}$  is dominated by motion from the south Galactic pole towards the North Galactic pole. This is towards the Great Wall in the northern Galactic hemisphere, while outflow towards the A569/P-P region ( $l \simeq 165^\circ$ ,  $-50^\circ < b < 30^\circ$ ) and the overdensity around C $\eta$  ( $l \simeq 240^\circ$ ,  $b \simeq -15^\circ$ ) can also be seen. Backside infall is now seen behind P-I-T and southern Centaurus. Finally, in figure 3d, at  $r = 8000 \; \mathrm{km \; s^{-1}}$ , we can see evidence of the mass distribution at larger radii. Beyond 10,000km s<sup>-1</sup>, this reconstruction places large superclusters at  $l \simeq 60^\circ$ ,  $b \simeq 60^\circ$  (forming out of the Great Wall), and  $l \simeq 220^\circ$ ,  $b \simeq -30^\circ$  (forming behind C $\eta$  and A400). These cause the observed outflow, while strong infall can be seen behind Pegasus ( $l \simeq 90^\circ$ ,  $b \simeq -25^\circ$ ) and Hydra ( $l \simeq 285^\circ$ ,  $b \simeq 5^\circ$ ).

Figure 4 shows the density field on planes at Supergalactic x, y, z = 0,  $\pm 4000 \text{ km s}^{-1}$ , while figure 5 shows the corresponding velocity field. The Supergalactic Plane can be clearly seen on the  $SGZ = 0 \text{ km s}^{-1}$  slice, with Centaurus and P-P visible as strong overdensities. The velocity field on the same plane shows the tug-of-war between these two superclusters, and the resultant effect on the local group. There has been some controversy over the presence of backside infall in the GA region (e.g. Dressler & Faber 1990, Mathewson, Ford & Buchhorn 1992). In this reconstruction, backside infall can clearly be seen for both Centaurus/GA (centred at  $SGX \simeq -3500 \text{km s}^{-1}$ ,  $SGY \simeq 1500 \text{km s}^{-1}$  in figure 4e) and P-P (centred at  $SGX \simeq 5500 \text{km s}^{-1}$ ,  $SGY \simeq -1500 \text{km s}^{-1}$  in figure 4e). A cone including the Centaurus/Great Attractor region  $(305^{\circ} < l < 325^{\circ}$ ,  $10^{\circ} < b < 25^{\circ}$ ) is shown in §6 as figure 9c. This shows velocity turnaround at  $r \simeq 3500 \text{ km s}^{-1}$ , with line-of-sight infall velocity reaching a maximum of  $\sim 300 \text{ km s}^{-1}$  at  $r \simeq 4500 \text{ km s}^{-1}$ .

## 5. Effects of Parameters on Reconstruction

In order to assess the effects of the parameters discussed in §3, the reconstruction was also carried out with a variety of different parameter values. When expanding the initial, redshift-space density field in terms of harmonics, the expansion at some radial and angular mode. This determines the resolution of the initial expansion, and hence the final reconstruction, as the whole procedure is carried out in terms of these same harmonics. If the expansion is terminated at a very low order, structures on a smaller scale than wavelength of the highest harmonic will be lost. On the other hand, if the expansion is continued to a very high order, we risk introducing artefacts as the density field ripples on scales smaller than the real structures. Appendix B of FLHLZ contains a detailed discussion of these problems.

Figure 6 shows the density field on the  $SGZ=0~{\rm km~s^{-1}}$  plane, demonstrating how additional structure is resolved as angular modes are added. In figure 6a, the angular modes are terminated at  $l_{\rm max}=4$ , and the circular structures associated with low-frequency angular modes are very clear. Overdensities occur on distinct shells, and follow circular paths about the origin; furthermore, resolution falls with radius, as the wavelength of the highest order mode increases. In figure 6b,  $l_{\rm max}=10$ , and the structures are much more detailed, with less obvious artefacts. The arc across the top right still follows a circular path, and do the two clusters at the bottom left. Nonetheless, structure is resolved quite well, with both the P-P and Centaurus/GA regions clearly visible. Comparing these to the reconstructions earlier in this paper, where  $l_{\rm max}=15$ , it is interesting to note that the purely angular arc across the top left of the  $SGZ=0~{\rm km~s^{-1}}$  slice is still present.

Beyond those parameters inherent in the expansion, a choice of cosmological parameters is necessary for the actual reconstruction. The shape of the prior power spectrum, characterised by  $\Gamma$  and  $\sigma_8$ , seems not to affect the reconstruction strongly within  $r < 10,000 \; \mathrm{km \; s^{-1}}$ . We have also performed the reconstruction of the IRAS velocities with a standard CDM ( $\Gamma = 0.5$ ) prior. Since the standard CDM model has less large scale power than the  $\Gamma = 0.2$  model, the WF smoothes more on large scales and therefore the reconstructed velocities tend to be smaller; the overall difference is however small with  $\langle Delta_{\rm RMS} \rangle \lesssim 50 \; \mathrm{km \; s^{-1}}$ . Figure

7 shows density and velocity fields on the  $SGZ=0~{\rm km~s^{-1}}$  plane for two different prior power spectra;  $(\Gamma=0.2,\,\sigma_8=1.0)$  and  $(\Gamma=0.5,\,\sigma_8=0.7)$ . Although the choice of  $\beta$  plays a complex role in the mechanics of the reconstruction, changes within the range  $0.2<\beta<1.0$  have little effect other than a linear scaling of peculiar velocities. Figure 8 shows the density fields on the  $SGZ=0~{\rm km~s^{-1}}$  plane for  $\beta=0.5$  and  $\beta=1.0$ , and the residual differences in the density and velocity fields. The residual velocity field is calculated by correcting for linear velocity scaling such that  $|{\bf v}| \propto \beta$ . Hence:

$$\mathbf{v}_{\text{resid}} = \frac{\mathbf{v}_1}{\beta_1} - \frac{\mathbf{v}_2}{\beta_2} \quad . \tag{7}$$

As can be seen from the plots, there is very little discrepancy from this linear scaling, indicating that within this volume, the reconstruction is robust to changes in  $\beta$ .

# 6. Comparison with Mark III

The Mark III catalogue (Willick et al. 1995) contains about 3000 galaxy peculiar velocities from Tully-Fisher distances. In figure 9, we show comparisons between Mark III velocities and those reconstructed from the IRAS 1.2Jy survey. These are plotted for cones selected by Galactic latitude and longitude, following Faber & Burstein (1988). Throughout these plots, it is important to note that the Mark III data is unsmoothed and hence displays considerably more scatter than that from WF reconstruction. As such, it is not possible to make anything more than a qualitative comparison. Davis, Nusser & Willick (1996) recently made a mode-by-mode comparison of IRAS and Mark III using spherical harmonics, finding a reduced  $\chi^2 \sim 2$ . A summary of other comparisons is given in Dekel (1994).

Figure 9a shows the central region of the Virgo cluster  $(265^{\circ} < l < 315^{\circ}, 67^{\circ} < b < 80^{\circ})$ . There is good agreement on the velocity gradient at around 1500 km s<sup>-1</sup>, where data are rich in both catalogues. However, for 2000 km s<sup>-1</sup> < r < 3000 km s<sup>-1</sup>, the WF velocities are near zero, while Mark III shows a continuation of the earlier velocity gradient, giving strong backside infall  $(v_{\rm radial} \simeq -1200$  km s<sup>-1</sup> at  $r \simeq 2500$  km s<sup>-1</sup>). A similar effect is seen in figure 9b, which shows the Leo Cloud  $(200^{\circ} < l < 260^{\circ}, 50^{\circ} < b < 70^{\circ})$ . Velocity gradients in the two catalogues are very similar out to about 1500 km s<sup>-1</sup>, after which the WF levels out while the Mark III continues to show back-side infall out to  $r \simeq 3000$  km s<sup>-1</sup>; beyond this, Mark III data is dominated by scatter. The Centaurus/Great Attractor region  $(305^{\circ} < l < 325^{\circ}, 10^{\circ} < b < 25^{\circ})$  is shown in figure 9c. In both surveys clear back-side infall can be seen, with turn-around at  $r \simeq 3500$  km s<sup>-1</sup>. Apart from the expected scatter in the Mark III, there is very good agreement. Figure 9d shows the Fornax-Eridanus region  $(193^{\circ} < l < 245^{\circ}, -66^{\circ} < b < -46^{\circ})$ . Again, the Mark III data shows considerably higher velocity contrast than the WF reconstruction, with a  $\sim 1000$  km s<sup>-1</sup> velocity difference across the nearby cluster, compared to  $\sim 300$  km s<sup>-1</sup> for WF.

Many of these differences might well be inherent to the WF procedure, which, by construction, goes to  $v_{\rm pec}=0$  at large scales. In regions of sparse IRAS data, this could account for the reduced velocity contrast in the WF reconstruction. In principle, a comparison between reconstructed velocities and those from direct measurement could be used to constrain power from larger scales.

# 7. The Acceleration of the Local Group

The dipole temperature anisotropy of the microwave background (CMB) has been measured by the COBE satellite to extraordinary accuracy,  $D=3.343\pm0.016\,\mathrm{mK}$  in the direction ( $l=264.4^\circ\pm0.3^\circ$ ,  $b=48.4^\circ\pm0.5^\circ$ ) (Smoot et al. 1991, 1992; Kogut et al. 1993; Fixsen et al. 1994). Although various alternative theories have been proposed (e.g., Gunn 1988; Paczyński & Piran 1990; Langlois & Piran 1996), this anisotropy is usually interpreted as due to the motion of the Earth with respect to a rest frame defined by the CMB. After correcting the Earth's motion relative to the Local Group (LG) barycenter, one infers (Smoot et al. 1991, Kogut et al. 1993) that the LG is moving at a velocity of  $627\pm22\,\mathrm{km~s^{-1}}$  in a direction ( $l=276^\circ\pm3^\circ$ ,  $b=30^\circ\pm3^\circ$ ).

In the gravitational instability scenario, the peculiar velocity of the LG is generated by surrounding fluctuations in the mass density. In linear theory, the relation between the gravitational acceleration induced by the mass inhomogeneities and the LG dipole is particularly simple,

$$\mathbf{v}(\mathbf{0}) = \frac{f(\Omega)H_0}{4\pi} \int d^3\mathbf{r} \,\delta(\mathbf{r}) \,\frac{\hat{\mathbf{r}}}{r^2} \quad . \tag{8}$$

In the context of the linear biasing model adopted in our reconstruction procedure, the the integral over the mass fluctuations can be replaced by an integral over the galaxy fluctuations provided we allow for a relative bias, i.e.,  $f(\Omega) \to \beta = f(\Omega)/b$ . Note the dipole is independent of the extragalactic distance scale if we measure distances in km s<sup>-1</sup>.

The integral in equation 8 can be evaluated using a galaxy redshift survey. By comparing the result with the known LG velocity, one infers an estimate of  $\beta$ . This technique (and variants using a flux-weighted dipole) has been applied by many authors to both optical and infrared galaxy samples as well as clusters catalogues (see the review by Strauss & Willick and references therein). The main uncertainty in the value of  $\beta$  determined in analyses of the dipole stem from statistical noise due to sparse sampling, the effects of redshift distortion, and the unknown contribution to the dipole from scales larger than the sample size or structures hidden behind the Zone of Avoidance.

The dipole predicted from equation 8 is (with our prescription of linear bias) linear in the galaxy density field. Consequently, the dipole computed from our WF algorithm will be an optimal (in the sense of minimum variance) estimate of the dipole due to matter within the reconstruction volume to the extent that non-linear effects can be neglected. Figure 10 shows the predicted dipole generated by matter within radius R as a function of R as computed from our WF reconstruction (see Appendix A) for our canonical prior ( $\Gamma = 0.2$ ,  $\sigma_8 = 0.7$ , and  $\beta = 0.7$ ). The amplitude of reconstructed dipole for the model is slightly larger than the dipole observed by COBE. The amplitude reconstructed dipole is roughly proportional to  $\beta$  (there is some non-linear dependence which enters in the correction for the redshift distortion, see equations 16 and 18 of FLHLZ) and therefore a reconstruction with value of  $\beta$  slightly smaller than our canonical model would provide a better fit. The direction of the LG dipole is shown in figure 11. The misalignment angle between the IRAS and CMB Local Group dipoles is only 13° out to  $R = 50 h^{-1}$ Mpc, partially due to the 'tug-of-war' between the Centaurus/GA and P-P superclusters. However, at  $R = 200 h^{-1}$ Mpc, the dipole of our canonical reconstruction points in the direction ( $l = 247^{\circ}, b = 37^{\circ}$ ) which is 25° away from the COBE direction. As our procedure suppresses shot-noise this misalignment may indicate non-linear effects on these very large scales. However, assessing the significance of this misalignment is difficult, as the dipole observed from any volume-limited sample will inherently show scatter about the 'true' CMB value (Vittorio & Juszkiewicz 1987, Juszkiewicz, Vittorio & Wyse 1990, Lahav, Kaiser & Hoffman 1989). For example, Lahav et al. (1989) show that for a biased CDM universe, 85% of observers will see a misalignment angle of  $\theta < 20^{\circ}$  in a sample out to  $r = 4000 \text{ km s}^{-1}$ .

In figure 11, we show the scatter in the reconstructed dipole due statistical noise and from contributions from fluctuations outside our reconstruction volume. The statistical scatter about the dipole can be computed within the framework of the WF algorithm and the result for the scatter in the dipole is given in Appendix B. This scatter is shown in figure 11 as the set dotted curves; this scatter grows with distance as the smoothing by the WF increases to mitigate the effects of shot noise. The uncertainty in the dipole due to matter outside the reconstruction volume can be calculated given the power spectrum assumed in the prior; the result is given in Appendix B and shown (for our canonical prior) as the dashed lines in figure 11. This scatter decreases steadily to zero as R increases. We should point out that the dipole generated within r < R is a well defined quantity.

## 8. Bulk Velocities

A simple statistic of the peculiar velocity field is the average or bulk velocity within a window,  $W(\mathbf{r})$ ,

$$\langle \mathbf{v} \rangle_{R_s} = \int_{r < R_s} d^3 \mathbf{r} \, \mathbf{v}(\mathbf{r}) \, W(\mathbf{r}) \quad .$$
 (9)

A commonly used window is spherical top-hat,  $W(r) = (4\pi/3R^3)^{-1}\Theta(r-R)$ , where  $\Theta(x)$  is the usual step function. The bulk velocity is sensitive to the fluctuations on scales > R and thus probes the linear regime even when non-linear effects in the density field on scales < R are important.

It is straightforward to relate the bulk velocity of spherical window centred on the LG to the expansion of the density field in spherical harmonics (see Appendix C). Once again, the WF reconstructed velocity field can be used to compute an optimal estimate of the bulk velocity in linear theory. As in the dipole calculation there will be scatter introduced by dilute sampling and fluctuations outside the reconstruction which can be computed for a given prior model of power spectrum (see Appendix C).

In figure 12, we show the Cartesian components of the bulk flow computed within spherical windows of radius, R. The points with error bars show the WF reconstruction. The curves show the values from the Potent analysis (Dekel 1994 and private communication) of the Mark III compilation of peculiar velocities (Willick et al. 1995). The reconstructed IRAS bulk flow out to 5000 km s<sup>-1</sup> is  $\sim 300$  km s<sup>-1</sup>, which agrees in amplitude with that derived from the Mark III peculiar velocities ( $\sim 370$  km s<sup>-1</sup>). However, the two vectors deviate by  $\sim 70^{\circ}$  in direction; there is a 33 percent probability of two random vectors aligning within  $70^{\circ}$ . This discrepancy could be due to differences in the width of the zone of avoidance and other systematic effects.

## 9. The Extent of the Supergalactic Plane

The so-called Supergalactic Plane (SGP) was recognised by de Vaucouleurs (1956) using the Shapley-Ames catalogue. This followed earlier work by Vera Rubin, whose analysis of radial velocities of nearby galaxies suggested a differential rotation of the 'metagalaxy'. Indeed, the SGP had in fact already been noted by William Herschel more than 200 years earlier. Traditionally the Virgo cluster was regarded as the centre of the Supergalaxy, and this was termed the 'Local Supercluster'. However, recent maps (e.g. figures 2-5) of the local universe indicate that much larger clusters (e.g. Great Attractor, Perseus-Pisces) are major components of this 'plane'. The north pole of the standard SGP (de Vaucouleurs 1976) lies in the direction of Galactic coordinates ( $l = 47.37^{\circ}$ ,  $b = 6.32^{\circ}$ ). The origin of SGL is at ( $l = 137.37^{\circ}$ ,  $b = 0^{\circ}$ ), which is one of the two regions where the SGP is crossed by the Galctic Plane. The Virgo cluster is at SGP coordinates ( $SGL = 104^{\circ}$ ,  $SGB = -2^{\circ}$ ).

Although the SGP is clearly visible in whole-sky galaxy catalogues, it has only been re-examined quantitatively in recent years. Tully (1986) claimed that the flattened distribution of clusters extended across a diameter of  $\sim 0.1c$  with axial ratios of 4:2:1. Shaver & Pierre (1989) found that radio galaxies were more strongly concentrated towards the SGP than were optical galaxies, and that the SGP (as represented by radio galaxies) extended out to redshift  $z \sim 0.02$ . Di Nella & Paturel (1995) examined the SGP using a compilation of nearly 5700 galaxies larger than 1.6 arcmin, and found qualitatively agreement with the standard SGP. Lahav et al. (1996, in preparation; also Lahav 1996) revisited the SGP using the Optical Redshift Survey (Santiago et al. 1995) and the IRAS 1.2 Jy survey (Fisher et al. 1995) To objectively identify a 'plane' they calculated the moment of inertia (with the observer located at the centre) by direct summation over the galaxies (taking into account the selection function) and subtracting the mean background density  $n_{\rm bg}$  in the absence of the 'plane'. By finding the eigenvalues and vectors of the inertia tensor they deduced that the derived 'plane' is aligned to within 30° of the standard SGP. The diameter of the SGP in ORS and IRAS was estimated to be at least 12,000 km s<sup>-1</sup>.

Here we take a different approach as the reconstructed field is continuous. Again, we can use our formalism to obtain the optimal reconstruction (in the minimum variance sense) of the moment of inertia. As explained

earlier, a convenient property of this Wiener approach applied to the density fluctuation field,  $\delta$ , is that it will also give the optimal reconstruction for any property which is linear in  $\delta$ . In particular, if we seek the optimal reconstruction of the moment of inertia

$$\tilde{C}_{ij} = C_{ij} - \bar{C}_{ij} = \frac{1}{N} \iiint \left[ \rho \left( \mathbf{x} \right) - n_{\text{bg}} \right] \left( x_i - \bar{x}_i \right) \left( x_j - \bar{x}_j \right) dV \qquad (10)$$

This can be re-written as

$$\tilde{C}_{ij} = \left(\frac{3}{4\pi R^3}\right) I_{ij} + \left(\left[1 - \frac{n_{\text{bg}}}{\bar{\rho}}\right] \frac{R^2}{5}\right) \delta_{ij}^k \qquad , \tag{11}$$

where  $\delta^k$  is the Kroneker delta and

$$I_{ij} = \int_{R} \delta(\mathbf{r}) x_i x_j dV \qquad . \tag{12}$$

This can be expressed analytically in terms of the reconstructed coefficients  $\delta_{lmn}^R$ . The full mathematical details are given in Appendix D. We emphasise again that in the Wiener approach the density field goes to the mean density at large distances. This does not necessarily mean that the SGP itself disappears at large distances, it only reflects our ignorance of what exists out there, where only very poor data are available.

To find the alignment and extent of the 'plane', we diagonalise the covariance matrix and find the eigenvalues and eigen-vectors :

$$\tilde{C}\mathbf{u}_{\alpha} = \lambda_{\alpha}\mathbf{u}_{\alpha}$$
 , (13)

where the  $\lambda_{\alpha}$ 's and  $\mathbf{u}_{\alpha}$ 's are the eigen-values and eigen-vectors respectively ( $\alpha = 1, 2, 3$ ). The 'half-width' (1-sigma) along each of the 3 axes is given by  $\sqrt{\lambda_{\alpha}}$ . Note that since the background contribution (the last term in equation 11) is isotropic, it only affects the eigen-values, but not the directions of the eigen-vectors.

We have applied this technique to the reconstructed, real-space density field, assuming as priors  $\beta = 0.7$ ,  $\sigma_8 = 0.7$  and a CDM power spectrum with  $\Gamma = 0.2$ . Out to  $R_{\text{max}} = 4000, 6000, 8000 \text{ km s}^{-1}$  the derived plane is aligned with de Vaucouleurs' SGP to within 15°, 35° and 31° respectively. Note that the probability of 2 random vectors being aligned within an angle  $\theta$  is

$$P(<\theta) = \frac{1 - \cos(\theta)}{2} \qquad . \tag{14}$$

Hence, for  $\theta = 30^{\circ}$  only  $P \sim 7\%$ .

Out to  $R_{\text{max}} = 4000 \text{ km s}^{-1}$  the axial ratio is roughly 7:5:1, assuming a background ratio  $(n_{\text{bg}}/\bar{\rho}) = 0.79$ , derived from the IRAS data. The results are only slightly different for raw harmonics, uncorrected for redshift distortion and noise, suggesting that redshift distortion is negligible on these large scales. The results also agree well with those direct summation of the moment of inertia (Lahav *et al.* 1996, in preparation).

#### 10. Conclusions & Discussion

In this paper, we have applied Wiener reconstruction with spherical harmonics and Bessel functions to the IRAS 1.2Jy redshift survey. Using a prior based on parameters  $\beta = 0.7$ ,  $\sigma_8 = 0.7$  and a CDM power spectrum with  $\Gamma = 0.2$ , we find that:

- 1. The reconstructed density field clearly shows many known structures, including N1600 (Santiago et al. 1995) and A3627 (Kraan-Korteweg et al. 1996). Notably, Perseus-Pisces is seen to extend out to  $R \sim 9000~\rm km~s^{-1}$ , while Virgo, Centaurus and Telescopium-Indus-Pavonis join to form a large structure extending over 1500 km s<sup>-1</sup> <  $R < 7000~\rm km~s^{-1}$ . A number of new clusters are also observed. Detailed maps are shown in §4.
- 2. The reconstructed velocity field shows backside infall for both Perseus-Pisces and the Centaurus/Great Attractor region. The Centaurus/Great Attractor supercluster shows velocity turnaround

at  $r \simeq 3500 \ \rm km \ s^{-1}$ , with line-of-sight infall velocity reaching a maximum of  $\sim 300 \ \rm km \ s^{-1}$  at  $r \simeq 4500 \ \rm km \ s^{-1}$ . Further, there is reasonable qualitative agreement between the reconstructed velocity field and that derived from Tully-Fisher measurements (Mark III).

- 3. The misalignment angle between the CMB and reconstructed IRAS LG dipoles falls to a minimum of  $13^{\circ}$ , calculated for  $R < 50 \ h^{-1}{\rm Mpc}$ , but increases to  $25^{\circ}$  for  $R < 200 \ h^{-1}{\rm Mpc}$ .
- 4. The reconstructed IRAS bulk flow out to 5000 km s<sup>-1</sup> is  $\sim 300$  km s<sup>-1</sup>, which agrees in amplitude with that derived from the Mark III peculiar velocities ( $\sim 370$  km s<sup>-1</sup>). However, the two bulk flow vectors deviate by some  $\sim 70^{\circ}$ .
- 5. The alignment and extent of the reconstructed Supergalactic Plane can be determined analytically from the harmonic coefficients. Out to  $R_{\rm max}=4000,6000,8000~{\rm km~s^{-1}}$  the derived plane is aligned with de Vaucouleurs' SGP to within 15°, 35° and 31° respectively. Out to  $R_{\rm max}=4000~{\rm km~s^{-1}}$  the axial ratio is roughly 7:5:1.

We also confirm that the reconstruction is robust to changes in prior parameters.

Wiener reconstruction is particularly well suited to recovering real-space density and velocity fields from near whole-sky surveys. The WF formalism provides a rigorous methodology for variable smoothing, determined by the sparseness of data relative to the expected signal. As such, natural continuations of this work would be application to newer surveys, such as the PSCZ (Saunders *et al.*, in preparation), which is complete down to 0.6Jy, or ORS (Santiago *et al.* 1995).

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#### Appendices

## Appendix A: Wiener Filtering

Here, we give a brief review of the Wiener Filter technique; the reader is referred to LFHSZ, ZHFL, and Rybicki & Press (1992) for further details. Let us assume that we have a set of measurements,  $\{d_{\alpha}\}$  ( $\alpha = 1, 2, ... N$ ) which are a linear convolution of the true underlying signal,  $s_{\alpha}$ , plus a contribution from statistical noise,  $\epsilon_{\beta}$ , such that

$$d_{\alpha} = \mathcal{R}_{\alpha\beta} \left[ s_{\beta} + \epsilon_{\beta} \right] \qquad , \tag{A1}$$

where  $\mathcal{R}_{\alpha\beta}$  is the response or "point spread" function (summation convention assumed). In the present context it would be the radial coupling matrix discussed in the previous section. Notice that we have assumed that the statistical noise is present in the underlying field and therefore is convolved by the response function.

The WF is the *linear* combination of the observed data which is closest to the true signal in a minimum variance sense. More explicitly, the WF estimate is given by  $s_{\alpha}(WF) = F_{\alpha\beta} d_{\beta}$  where the filter is chosen to minimise  $\langle |s_{\alpha}(WF) - s_{\alpha}|^2 \rangle$ . It is straightforward to show (see ZHFL for details) that the WF is given by

$$F_{\alpha\beta} = \langle s_{\alpha} d_{\gamma} \rangle \langle d_{\gamma} d_{\beta}^{\dagger} \rangle^{-1} \qquad , \tag{A2}$$

where

$$\langle s_{\alpha} d_{\beta}^{\dagger} \rangle = \mathcal{R}_{\beta \gamma} \langle s_{\alpha} s_{\gamma}^{\dagger} \rangle \tag{A3}$$

$$\langle d_{\alpha} d_{\beta}^{\dagger} \rangle = \mathcal{R}_{\alpha \gamma} \, \mathcal{R}_{\beta \delta} \, \left[ \langle s_{\gamma} s_{\delta}^{\dagger} \rangle + \langle \epsilon_{\gamma} \epsilon_{\delta}^{\dagger} \rangle \right] \qquad . \tag{A4}$$

In the above equations, we have assumed that the signal and noise are uncorrelated. From equation A4, it is clear that in order to implement the WF one must construct a *prior* which depends on the variance of the signal and noise.

The dependence of the WF on the prior can be made clear by defining signal and noise matrices given by  $S_{\alpha\beta} = \langle s_{\alpha}s_{\beta}^{\dagger} \rangle$  and  $N_{\alpha\beta} = \langle \epsilon_{\alpha}\epsilon_{\beta}^{\dagger} \rangle$ . With this notation, we can rewrite equation A4 as

$$\mathbf{s}(WF) = \mathbf{S} \left[ \mathbf{S} + \mathbf{N} \right]^{-1} \mathcal{R}^{-1} \mathbf{d} \qquad . \tag{A5}$$

Formulated in this way, we see that the purpose of the WF is to attenuate the contribution of low signal to noise ratio data and therefore regularize the inversion of the response function. The derivation of the WF given above follows from the sole requirement of minimum variance and requires only a model for the variance of the signal and noise. The WF can also be derived using the laws of conditional probability if the underlying distribution functions for the signal and noise are assumed to be Gaussian; in this more restrictive case, the WF estimate is, in addition to being the minimum variance estimate, also both the maximum a posterior estimate and the mean field (see LFHSZ, ZHFL). For Gaussian fields, the mean WF field can be supplemented with a realisation of the expected scatter about the mean field to create a realisation of the field; this is the heart of the "constrained realisation" approach described in Hoffman & Ribak (1991; see also ZHFL).

As Rybicki & Press (1992) point out, the WF is in general a biased estimator of the mean field unless the field has zero mean; this is not a problem here since we will perform the filtering on the density fluctuation field which has, by construction, zero mean.

## Appendix B: Dipole Velocity in Spherical Harmonics

The velocity of the central observer in the CMB frame, commonly referred to a "dipole", due to the fluctuations with r < R can be related to the spherical harmonic expansion of the density field (see FLHLZ equation C9),

$$\mathbf{v}(\mathbf{0})_{r < R} = \frac{\Omega^{0.6}}{4\pi} \int_{V_R} d^3 \mathbf{r}' \, \delta(\mathbf{r}') \, \frac{\mathbf{r}'}{r'^3}$$

$$= \frac{\beta}{4\pi} \sum_{lmn} C_{ln} \frac{\delta_{lmn}^R}{k_n} \int_0^R dr' \, j_l(k_n r') \int_{4\pi} d\Omega \, \hat{\mathbf{r}}' \, Y_{lm}(\hat{\mathbf{r}}')$$

$$= \frac{\beta}{4\pi} \left( \frac{4\pi}{3} \right)^{1/2} \sum_n \frac{C_{1n}}{k_n} \left( -\sqrt{2} Re[\delta_{11n}^R] \, \hat{\mathbf{x}} + \sqrt{2} Im[\delta_{11n}^R] \, \hat{\mathbf{y}} + Re[\delta_{10n}^R] \, \hat{\mathbf{z}} \right) \, (1 - j_0(k_n R)) .$$
(B1)

Here Re[a] and Im[a] refer to the real and imaginary parts of a complex number, a. The last line provides a convenient expression for the Cartesian components of the dipole. In the limit that  $R \to \infty$ , we recover the true dipole, apart from statistical noise and systematic errors introduced by non-linear effects. From the last line equation (B1), we see that the dipole within r < R can thus be regarded as the sum of two terms: the true CMB dipole and a correction for finite sample size (the  $j_0(k_n R)$  term). In fact the latter term involving the  $j_0(k_n R)$  can be shown to be the velocity of a shell at a distance R in the CMB frame.

There are two main sources of error in the dipole derived via equation (B1). First, there is a statistical noise introduced by the finite sampling of the density field by the IRAS catalogue. The scatter in the reconstructed dipole can be easily calculated in the framework of the Wiener filter (see FLHLZ Appendix F for the scatter in the reconstructed density and velocity field). The scatter in each component of the dipole is given by

$$\langle \Delta \mathbf{v}(\mathbf{0}) \rangle_{\text{wf}}^2 = \frac{\beta^2}{12\pi} \sum_{nn'} C_{1n} C_{1n'} \frac{(1 - j_0(k_n r)) (1 - j_0(k_{n'} r))}{k_n k_{n'}} [(\mathbf{I} - \mathbf{F}_1) \mathbf{S}_1]_{nn'} , \qquad (B2)$$

where  $\mathbf{F}_l = \mathbf{S}_l(\mathbf{S}_l + \mathbf{N}_l)^{-1}$  is defined in terms of the signal,  $\mathbf{S}_l$ , and noise,  $\mathbf{N}_l$  matrices.

The second source of error is from fluctuations outside the volume used in the Wiener reconstruction. The contribution to the dipole from r > R,

$$\mathbf{v}_{out}(\mathbf{0}) = \frac{\beta}{4\pi} \int_{r < R} d^3 \mathbf{r}' \, \delta(\mathbf{r}') \frac{\mathbf{r}'}{r'^3} \quad , \tag{B3}$$

introduces a 1-D rms scatter in the dipole

$$\langle \Delta \mathbf{v}(\mathbf{0}) \rangle_{out}^2 = \frac{1}{3} \frac{\beta^2}{2\pi^2} \int dk \, P(k) \left[ j_o(kR) \right]^2 \quad . \tag{B4}$$

In the above equation, P(k) is the linear power spectrum defined by the prior. The total error in the reconstructed dipole is taken to be the quadrature sum of equations B3 and B4.

## Appendix C: Bulk Velocity in Spherical Harmonics

The average or "bulk" velocity measured within a spherically symmetric region defined by window function with characteristic scale  $r = R_s$  is defined by

$$\langle \mathbf{v} \rangle_{R_s} = \int d^3 \mathbf{r} W(r) \mathbf{v}(\mathbf{r}) \quad , \tag{C1}$$

where the volume integral of the window is taken to be unity. If we rewrite the above expression in terms of the Fourier components of  $\mathbf{v}$ ,

 $\langle \mathbf{v} \rangle_{R_s} = \frac{1}{(2\pi)^3} \int d^3 \mathbf{k} \, \mathbf{v_k} \, W(kR_s)$ 

where  $W(kR_s)$  is the Fourier transform of the window function  $(W(kR_s) = 3j_1(kR_s)/kR_s)$  for spherical top-hat). From the above equation, we see that the bulk velocity for region centred on the LG is formally equivalent to the dipole velocity of LG induced by the density field obtained from smoothing with the window. Consequently, the analogous formulae for the bulk velocity, uncertainty due to sampling, and the contributions from fluctuations outside the reconstruction volume in terms of the spherical harmonics are readily derived by substituting

$$\delta_{lmn} \to \delta_{lmn} W(k_n R_s)$$
 and  $P(k) \to P(k) W(k_n R_s)$ 

in equations B1, B2, and B4

# Appendix D: Moment of Inertia in Spherical Harmonics

Consider the moment of inertia tensor within a sphere of radius R around the origin:

$$I_{ij} = \int_{R} \delta(\mathbf{r}) x_i x_j dV . \tag{1}$$

We now express the density fluctuation in terms of spherical harmonics and Bessel functions:

$$\delta(\mathbf{r}) = \sum_{l}^{l_{\text{max}}} \sum_{m=-l}^{m=+l} \sum_{n=1}^{n_{\text{max}}(l)} C_{ln} \, \delta_{lmn} \, j_l(k_n r) \, Y_{lm}(\hat{\mathbf{r}}) , \qquad (2)$$

By substituting eq. (2) into eq. (1) and utilising the properties of the spherical harmonics we find after some algebra the elements of the symmetric matrix  $I_{ij}$ :

$$I_{xx} = \sum_{n=1}^{n_{\text{max}}(2)} C_{2,n} B_{2,n} \left[ \sqrt{\frac{2\pi}{15}} \left( \delta_{2,-2,n} + \delta_{2,2,n} \right) - \sqrt{\frac{4\pi}{45}} \ \delta_{2,0,n} \right] + \frac{\sqrt{4\pi}}{3} \sum_{n=1}^{n_{\text{max}}(0)} C_{0,n} B_{0,n} \delta_{0,0,n}$$

$$I_{yy} = \sum_{n=1}^{n_{\text{max}}(2)} C_{2,n} B_{2,n} \left[ -\sqrt{\frac{2\pi}{15}} \left( \delta_{2,-2,n} + \delta_{2,2,n} \right) - \sqrt{\frac{4\pi}{45}} \, \delta_{2,0,n} \right] + \frac{\sqrt{4\pi}}{3} \sum_{n=1}^{n_{\text{max}}(0)} C_{0,n} B_{0,n} \delta_{0,0,n}$$

$$I_{zz} = \frac{\sqrt{4\pi}}{3} \left[ \frac{2}{\sqrt{5}} \left( \sum_{n=1}^{n_{\text{max}}(2)} C_{2,n} B_{2,n} \delta_{2,0,n} \right) + \left( \sum_{n=1}^{n_{\text{max}}(0)} C_{0,n} B_{0,n} \delta_{0,0,n} \right) \right]$$

$$I_{xy} = -i\sqrt{\frac{2\pi}{15}} \sum_{n=1}^{n_{\text{max}}(2)} C_{2,n} B_{2,n} (\delta_{2,-2,n} - \delta_{2,2,n})$$

$$I_{xz} = \sqrt{\frac{2\pi}{15}} \sum_{n=1}^{n_{\text{max}}(2)} C_{2,n} B_{2,n} (\delta_{2,-1,n} - \delta_{2,1,n})$$

$$I_{yz} = -i\sqrt{\frac{2\pi}{15}} \sum_{n=1}^{n_{\text{max}}(2)} C_{2,n} B_{2,n} (\delta_{2,-1,n} + \delta_{2,1,n})$$

where  $n_{\text{max}}(l)$  is the  $n_{\text{max}}$  corresponding to a particular value of l, and

$$B_{ln} \equiv \int_0^R j_l(k_n r) r^4 dr \qquad .$$

Note that as the  $I_{ij}$  is quadratic in the coordinates, the only harmonics to appear are of l=0 and l=2.

#### Figure Captions

- Fig. 1 Scatter between velocities generated in Local Group and CMB frames, plotted as a function of  $\beta$  used in the reconstruction. The amplitude of the scatter is divided by  $\beta$  so as to remove the linear scaling of velocities with  $\beta$  inherent in reconstructions based on linear theory. A clear minimum can be seen at  $\beta \simeq 0.7$ .
- Fig. 2 The reconstructed density field, evaluated on thin shells at various real space distances, shown in Galactic Aitoff projection. Dashed lines show  $\delta < 0$ , and solid lines show  $\delta \ge 0$ , with contour spacing  $\Delta \delta = 0.1$ . (a) 2000 km s<sup>-1</sup>: Clusters marked are N5864, Virgo (Vir), Centaurus (Cen), Hydra (Hyd), Fornax- Doradus-Eridanus (FDE), Ursa Major (Urs), and  $C\alpha$ . Voids marked are Local Void (LV),  $V\alpha$  and  $V\beta$ . (b) 4000 km s<sup>-1</sup>: Clusters marked are Centaurus (Cen), Telescopium-Indus-Pavonis (TIP), Hydra (Hyd), Leo, Cancer (Can), N1600, Camelopardalis (Cam), Perseus-Pisces (P-P), Cetus (Cet),  $C\beta$ ,  $C\gamma$  and  $C\delta$ . Voids shown are Local Void (LV) and  $V\beta$ . (c) 6000 km s<sup>-1</sup>: A3627, Hydra (Hyd), Leo, Cancer (Can), Orion (Ori), A569, Perseus-Pisces (P-P), Pegasus (Peg), Telescopium-Indus-Pavonis (TIP),  $C\gamma$ ,  $C\delta$ ,  $C\epsilon$  and  $C\zeta$ . One void is marked  $V\gamma$ . (d) 8000 km s<sup>-1</sup>: Clusters marked are Great Wall (GW), A3627, Hydra (Hyd), Leo, A539, A779, A400, Coma (Com), Perseus-Pisces (P-P), Pegasus (Peg), Cygnus (Cyg),  $C\gamma$ ,  $C\zeta$  and  $C\eta$ . The continuation of  $V\gamma$  is shown.
- Fig. 3 The reconstructed velocity field, evaluated on thin shells at various real space distances, shown in Galactic Aitoff projection. These velocity fields correspond with the density fields shown in figure 2. Dashed lines show infall, and solid lines show outflow. The first solid line is for  $v_{\rm radial} = 0 \ \rm km \ s^{-1}$ , and contour spacing is  $\Delta v_{\rm radial} = 50 \ \rm km \ s^{-1}$ . (a) 2000 km s<sup>-1</sup>: Note the velocity dipole due to the Centaurus/GA region (towards  $l \simeq 300^{\circ}$ ,  $b \simeq 15^{\circ}$ ). (b) 4000 km s<sup>-1</sup>: Note outflow towards P-P ( $l \simeq 145^{\circ}$ ,  $b \simeq -20^{\circ}$ ), N1600 ( $l \simeq 195^{\circ}$ ,  $b \simeq -20^{\circ}$ ) and Cancer ( $l \simeq 190^{\circ}$ ,  $b \simeq 20^{\circ}$ ). (c) 6000 km s<sup>-1</sup>: Dominated by motion from the south Galactic pole towards the North Galactic pole, caused by outflow towards the Great Wall in the northern Galactic hemisphere. (d) 8000 km s<sup>-1</sup>: Outflow caused by large superclusters at  $l \simeq 60^{\circ}$ ,  $b \simeq 60^{\circ}$  (forming behind the Great Wall), and  $l \simeq 220^{\circ}$ ,  $b \simeq -30^{\circ}$  (forming behind  $C\eta$  and A400). Also, note strong infall can be seen behind Pegasus ( $l \simeq 90^{\circ}$ ,  $b \simeq -25^{\circ}$ ) and Hydra ( $l \simeq 285^{\circ}$ ,  $b \simeq 5^{\circ}$ ).
- Fig. 4 The reconstructed density field on thin slices at Supergalactic  $x, y, z = 0, \pm 4000 \,\mathrm{km\ s^{-1}}$ . Contours start at  $\delta = 0$  with spacing  $\Delta \delta = 0.25$ . Note the clear presence of the Supergalactic plane in the  $SGZ = 0 \,\mathrm{km\ s^{-1}}$  plot (e).
- Fig. 5 The reconstructed velocity field on thin slices at Supergalactic  $x, y, z = 0, \pm 4000 \text{ km s}^{-1}$ . Arrows show the projection of the local peculiar velocity onto the plane, with length giving the amplitude in accordance with the axis scale. These velocity fields correspond to the density fields shown in figure 4. Note clear backside infall towards both Centaurus/GA (centred at  $SGX \simeq -3500 \text{ km s}^{-1}$ ,  $SGY \simeq 1500 \text{ km s}^{-1}$  in (e)) and P-P (centred at  $SGX \simeq 5500 \text{ km s}^{-1}$ ,  $SGY \simeq -1500 \text{ km s}^{-1}$  in (e)).
- Fig. 6 Cone diagrams comparing the IRAS reconstructed velocity field with that derived from Tully-Fisher measurements (Mark III). Hollow triangles show the IRAS reconstruction, while filled squares show Mark III. (a) Central region of the Virgo cluster  $(265^{\circ} < l < 315^{\circ}, 67^{\circ} < b < 80^{\circ})$ . (b) Leo Cloud  $(200^{\circ} < l < 260^{\circ}, 50^{\circ} < b < 70^{\circ})$ . (c) Centaurus/Great Attractor region  $(305^{\circ} < l < 325^{\circ}, 10^{\circ} < b < 25^{\circ})$ . (d) Fornax-Eridanus region  $(193^{\circ} < l < 245^{\circ}, -66^{\circ} < b < -46^{\circ})$ . Note that the Mark III data is unsmoothed and hence displays considerably more scatter than that from WF reconstruction.
- Fig. 7 Demonstration of different reconstruction resolutions. The reconstructed density field, evaluated on a thin slice at  $SGZ=0~{\rm km~s^{-1}}$ . Contours start at  $\delta=0$  with spacing  $\Delta\delta=0.1$ . Figure 5e shows the corresponding plot for  $l_{\rm max}=15$ ; the default used throughout this paper. (a)  $l_{\rm max}=4$ : Clear circular patterns can be seen in the density field. (b)  $l_{\rm max}=10$ : Only a few circular artefacts remain, given the addition of higher-order harmonics.
  - Fig. 8 Effects of varying cosmological parameters  $\Gamma$  and  $\sigma_8$ . (a) and (c) show the reconstructed density

field, evaluated for a thin slice at  $SGZ=0~{\rm km~s^{-1}}$ . Contours start at  $\delta=0$  with spacing  $\Delta\delta=0.1$ . (b) and (d) show the reconstructed velocity field, evaluated for the same thin slice. Arrows show the projection of the local peculiar velocity onto the plane, with length giving the amplitude in accordance with the axis scale. Figures 5e and 6e show the corresponding plots for  $\beta=0.7,~\Gamma=0.2,~\sigma_8=0.7$ ; the canonical parameters used throughout this paper. (a) and (c)  $\beta=0.7,~\Gamma=0.5,~\sigma_8=0.7$ . (b) and (d)  $\beta=0.7,~\Gamma=0.2,~\sigma_8=1.0$ .

Fig. 9 — Effects of varying the cosmological parameter  $\beta$ . (a), (b) and (c) are density fields, evaluated for a thin slice at SGZ = 0 km s<sup>-1</sup>. Contours start at  $\delta = 0$  with spacing  $\Delta \delta = 0.1$ . (a) and (b) show reconstructed density fields for  $\beta$  of 0.5 and 1.0 respectively, with  $\Gamma = 0.2$  and  $\sigma_8 = 0.7$ . Figures 5e shows the corresponding plot for  $\beta = 0.7$ ,  $\Gamma = 0.2$ ,  $\sigma_8 = 0.7$ ; the canonical parameters used throughout this paper. (c) shows the residual density field after subtraction of (a) from (b), demonstrating the extremely small differences caused by change of  $\beta$ . Finally, (d) shows the residual velocity field, evaluated for the same thin slice. Arrows show the projection of the local peculiar velocity onto the plane, with length giving the amplitude in accordance with the axis scale.

Fig. 10 — The amplitude of the acceleration, or dipole motion, of the LG caused by the fluctuations within a sphere of radius R versus R. The heavy solid curve shows the dipole derived from the Wiener reconstruction with our canonical prior ( $\Gamma = 0.2$ ,  $\sigma_8 = 0.7$ , and  $\beta = 0.7$ ). The dotted curves show the expected scatter from finite sampling (see Appendix B) while the dashed curves show the scatter due to fluctuations outside the reconstruction volume (i.e.,  $r < 200 \ h^{-1}{\rm Mpc}$ ); the dot-dashed curve is the quadrature sum of both terms. The dotted horizontal line is the value of the LG dipole inferred by COBE, 627 km s<sup>-1</sup>.

Fig. 11 — The direction of the LG dipole. The crosses show the convergence of the direction of the reconstructed dipole; starting at the top of the plot the crosses give the direction (in Galactic coordinates) of the dipole within R (see Fig. 1) as R is increased in 1  $h^{-1}$ Mpcintervals. The direction of the dipole inferred from the COBE measurements, ( $l = 276^{\circ}$ ,  $b = 30^{\circ}$ ; Smoot et al. 1991) is denoted by the star. The circular curves denote angular separations from the COBE result in 10° increments.

Fig. 12 — The average or bulk velocity within a sphere of radius R centered on the Local Group a function of R. The four panels show the three Cartesian components and the scalar amplitude of the bulk flow. The points with error bars represent the bulk flows in the Wiener reconstruction with our canonical prior ( $\Gamma = 0.2$ ,  $\sigma_8 = 0.7$ , and  $\beta = 0.7$ ). The error bars are the scatter due to both the finite sampling and fluctuations outside the reconstruction volume (see Appendix C). The triangles connected by the curves represent the measurements from the POTENT reconstruction algorithm applied the Mark III peculiar velocity data (taken from Dekel 1994). The two curves represent two different weighting schemes and reflect the systematic uncertainty; the estimated random error is approximately 15% (Dekel 1994).